Contracted Temporal Equilibrium Logic

Pedro Cabalar[1][0000−0001−7440−0953], Thomas Eiter[2][0000−0001−6003−6345], and Davide Soldà[2][0000−0001−7535−5605]

[1] University of A Coruña, SPAIN
cabalar@udc.es
[2] Institute of Logic and Computation,
Technische Universität Wien, AUSTRIA
eiter@kr.tuwien.ac.at, davide.solda@tuwien.ac.at

1 Introduction

(Linear-time) Temporal Equilibrium Logic (TEL) [1] is a temporal extension of Equilibrium Logic [4], a nonmonotonic logic that characterizes the answer sets of logic programs. The semantics of TEL is based on selecting specific models of a theory within Temporal Here-and-There (THT), a temporal extension of the intermediate logic Here-and-There [3]. The selected models consist of traces that are considered *in equilibrium* (also referred to as *stable models* or *stable traces*) when a certain minimality condition is satisfied, thereby creating a non-monotonic entailment relation. In this work, we explore a selection process that not only enforces minimality but also reduces the number of transitions within a trace, effectively contracting the model. To this end, we introduce *contracted* THT and *contracted* TEL as logical extensions based on model selection.

Trace selection by length has been previously addressed in the literature. For instance, the authors in [5] focus on selecting models by identifying the shortest counterexamples for model-checking purposes. In planning contexts, ASP solvers often operate up to a predetermined plan-horizon to generate the shortest possible plans. Additional approaches involve minimization criteria applied to weighted atoms; for example, [2] addresses LTL over finite traces, among others. Rather than imposing an external selection function on stable models, we aim to develop, in the spirit of TEL, a logical framework that selects models based on intrinsic criteria. Let us consider now a motivating example:

Example 1. Suppose that, to move to the airport from our office, we may go by bus or take a taxi. If we go by bus, we must make two bus stops, bs_1 and bs_2 before arriving whereas, if we go by taxi, we always have to stop at a crossroad c. The number of transitions we may take between two stops is not predetermined. The following TEL theory is one possible simplified formalization of this example (recall that $\Box$, $\Diamond$, stand for always, eventually, and next time, respectively):

$bus \vee taxi$	(1)	$\Box(bs_2 \rightarrow \Diamond airport)$	(4)
$\Box(bus \rightarrow \Diamond bs_1)$	(2)	$\Box(taxi \rightarrow \Diamond c)$	(5)
$\Box(bs_1 \rightarrow \Diamond bs_2)$	(3)	$\Box(c \rightarrow \Diamond airport)$	(6)

The stable models of (1)-(6) follow two different patterns:

1. $\{bus\} \cdot \emptyset^* \cdot \{bs_1\} \cdot \emptyset^* \cdot \{bs_2\} \cdot \emptyset^* \cdot \{airport\} \cdot \emptyset^*$
2. $\{taxi\} \cdot \emptyset^* \cdot \{c\} \cdot \emptyset^* \cdot \{airport\} \cdot \emptyset^*$

where, in both cases, we may replace the last $\emptyset^*$ by $\emptyset^\omega$ (for infinite traces). The shortest stable model corresponds to $\{taxi\} \cdot \{c\} \cdot \{airport\}$, where we take the taxi and it arrives in the fastest possible way, without any delay in each trip segment. We claim that the stable models $\{taxi\} \cdot \emptyset^? \cdot \{c\} \cdot \emptyset^? \cdot \{airport\}$ and $\{bus\} \cdot \emptyset^? \cdot \{bs_1\} \cdot \emptyset^? \cdot \{bs_2\} \cdot \emptyset^? \cdot \{airport\}$, although longer, should be incomparably minimal as well, as it corresponds to the shortest trace we may get *when we decide to take the bus*.

2 Approach

Informally, a trace is a finite or infinite sequence of states. To compare two different traces we start introducing the concept of a *contractor* function μ, a mapping that transforms indices $i \in [0, \dots, \lambda)$ from an interval of length λ into new positions $\mu(i)$ inside an interval of length $\lambda' \leq \lambda$. Formally, let $\lambda, \lambda' \in \mathbb{N} \cup \{\omega\}$ be two trace lengths. A *contractor* function μ from λ to λ' is any surjective function of type $\mu : [0, \dots, \lambda) \to [0, \dots, \lambda')$ that satisfies $\mu(0) = 0$ and $\mu(i+1) \leq \mu(i) + 1$ for all $i \in [0, \dots, \lambda)$ and $i + 1 < \lambda$.

Definition 1. *Let* $\mathbf{H}$ *and* $\mathbf{T}$ *be two traces of lengths* $\lambda_h = |\mathbf{H}|$ *and* $\lambda_t = |\mathbf{T}|$ *respectively. We say that a contractor* μ *from* λ_t *to* λ_h *leads to a* c*THT-trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$*, when* $T_i \supseteq H_{\mu(i)}$ *for all* $i \in [0, \dots, \lambda_t)$.

Definition 2 (cTHT-satisfaction). *Let* $\mathbf{M}$ *be a* c*THT-trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$ *over alphabet* $\mathcal{A}$ *and let* $\lambda = |\mathbf{H}|$*. Then* $\mathbf{M}$ satisfies *a temporal formula* φ *at step* k*, written* $\mathbf{M}, k \models \varphi$*, if:*

1. $\mathbf{M}, k \models \top$ *and* $\mathbf{M}, k \not\models \bot$*;*
2. $\mathbf{M}, k \models p$ *if* $p \in H_k$ *for any atom* $p \in \mathcal{A}$*;*
3. $\mathbf{M}, k \models \varphi \wedge \psi$ *iff* $\mathbf{M}, k \models \varphi$ *and* $\mathbf{M}, k \models \psi$*;*
4. $\mathbf{M}, k \models \varphi \vee \psi$ *iff* $\mathbf{M}, k \models \varphi$ *or* $\mathbf{M}, k \models \psi$*;*
5. $\mathbf{M}, k \models \varphi \to \psi$ *iff* $\begin{cases} \mathbf{M}, k \not\models \varphi \text{ or } \mathbf{M}, k \models \psi \\ \langle \mathbf{T}, \mathbf{T}, id \rangle, j \not\models \varphi \text{ or } \langle \mathbf{T}, \mathbf{T}, id \rangle, j \models \psi \; \forall j \in \mu^-(k) \end{cases}$
6. $\mathbf{M}, k \models \varphi$ *iff* $k+1 < \lambda$ *and* $\mathbf{M}, k+1 \models \varphi$*;*
7. $\mathbf{M}, k \models \varphi \mathbf{U} \psi$ *iff* $\exists j \in [k, \dots, \lambda)$*, s.t.* $\mathbf{M}, j \models \psi$ *and* $\mathbf{M}, i \models \varphi \; \forall i \in [k, \dots, j)$*;*
8. $\mathbf{M}, k \models \varphi \mathbf{R} \psi$ *iff* $\forall j \in [k, \dots, \lambda)$*,* $\mathbf{M}, j \models \psi$ *or* $\exists i \in [k \dots, j]$ *s.t.* $\mathbf{M}, i \models \varphi$*.*
9. $\mathbf{M}, k \models \varphi$ *iff* $|\mu^-(k)| = 1$*,* $k+1 < \lambda$*, and* $\mathbf{M}, k+1 \models \varphi$

The following result lifts an essential property of THT to the contracted setting: that every formula that is satisfied by a THT-trace $\langle \mathbf{H}, \mathbf{T} \rangle$ must be satisfied by $\mathbf{T}$ viewed as LTL-interpretation. It reflects the intuitionistic view that when moving from a state $\mathbf{H}$ to a state $\mathbf{T}$ with more truth information, inferences made will be preserved.

Theorem 1 (Persistence). *For every* c*THT-trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$ *with* $\lambda = |\mathbf{H}|$ *and every* $k \in [0, \dots, \lambda)$*:* $\mathbf{M}, k \models \varphi$ *implies* $\mathbf{T}, \mu^-(k) \models \varphi$*.*

Definition 3 (cTEL). *A total* c*THT-trace* $\langle \mathbf{T}, \mathbf{T}, id \rangle$ *is a* contracted temporal equilibrium model *(or* c-stable model*) of a theory* Γ *if it is a model of* Γ *(that is* $\mathbf{T}, 0 \models \Gamma$ *in LTL) and there is no model* $\langle \mathbf{H}, \mathbf{T}, \mu \rangle$ *of* Γ *with* $\mathbf{H} \neq \mathbf{T}$*.*

The desired (contracted) stable models of Example 1 are captured by Definition 3.

References

1. F. Aguado, P. Cabalar, M. Diéguez, G. Pérez, and C. Vidal. Temporal equilibrium logic: a survey. *Journal of Applied Non-Classical Logics*, 23(1-2):2–24, 2013.
2. Carmine Dodaro, Valeria Fionda, and Gianluigi Greco. LTL on weighted finite traces: Formal foundations and algorithms. In Luc De Raedt, editor, *Proceedings of the Thirty-First International Joint Conference on Artificial Intelligence, IJCAI 2022, Vienna, Austria, 23-29 July 2022*, pages 2606–2612. ijcai.org, 2022.
3. A. Heyting. Die formalen Regeln der intuitionistischen Logik. *Sitzungsberichte der Preussischen Akademie der Wissenschaften. Physikalisch-mathematische Klasse*, 1930.
4. David Pearce. Equilibrium logic. *Annals of Mathematics and Artificial Intelligence*, 47(1-2):3, 2006.
5. Viktor Schuppan and Armin Biere. Shortest counterexamples for symbolic model checking of ltl with past. In *International Conference on Tools and Algorithms for the Construction and Analysis of Systems*, pages 493–509. Springer, 2005.